The Method of Secondary Cycles for assessing the fatigue damage of a multiaxial stress-history

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ABSTRACT

An important issue that arises when applying a multiaxial rainflow, is how to post-treat the cycles extracted by the rainflow itself in a way that somehow takes into account their actual path when calculating their fatigue damage. Numerous approaches can be found in literature, proposed from different researchers, about the correction of the stress-amplitude of cycles extracted from rainflow-counting. This paper presents a new methodology called Method of Secondary Cycles.

KEYWORDS

Multiaxial fatigue, multiaxial rainflow counting, non-proportional loading, equivalent stress range.

INTRODUCTION

Broadly speaking, a multiaxial rainflow-counting is a tool that extracts a set of multiaxial cycles from a stress-history. A path in the stress space defines each of these cycles. The definition of the amplitude of such cycles, used to calculate their fatigue damage, is not unique and has been subject of research in the field of multiaxial fatigue. Since the pioneering work of Dang Van in 1973, Ref. [1], researchers have developed several methodologies aimed at assigning to a stress-cycle having a shape other than a segment in the stress space, an "equivalent amplitude" which takes into account in some way the actual load path.

These methodologies are essentially "geometrical", in the sense that they define somehow a measure of the equivalent stress amplitude based on geometrical characteristics of the stress path defining the fatigue cycle.

Conversely, the Method of Secondary Cycles proposed in this paper, is based on the creation of new subcycles nested in the primary stress path of the extracted cycle. Its theoretical validation is performed via an example.

THE MULTIAXIAL RAINFLOW

Before presenting the Method of Secondary Cycles for the correction of the stress cycles, it is necessary to describe synthetically how an algorithm of multiaxial rainflow works. The one this paper refers to is derived from the work of Wang and Brown, Ref. [2], and is the basis of the commercial software *MultiFat*, Ref. [3].

This methodology, starts from the original Wang and Brown's algorithm but it is adapted to work with any formulation of mean stress correction and type of S-N curve, thus making it easy to use with the curves available in the literature or deduced by bench tests.

It is worth remembering that when the stress story is purely uniaxial, the method is equivalent to Matsuishi and Endo's classic Rainflow, Ref. [4].

Below is a brief description of the Rainflow multiaxial algorithm, assuming that the fatigue failure criterion is the Von Mises stress. Let us consider a given stress sequence.

M_1) The sequence is firstly closed and then is reordered by placing as the first point the one with the maximum Von Mises stress σ_{VM} in the entire stress history

M_2) A cycle count begins starting from the first point, and then it will be repeated at each point in the sequence. The value of Von Mises stress relative to the starting point of the count is considered as the stress parameter, called $\sigma_{VM,REL}$. This is actually the Von Mises stress calculated on the tensor difference between the current point and the starting point of the count.

M_3) The final point of each counting cycle is obtained when the highest $\sigma_{VM,REL}$ in the current calculation is reached, or when the stress point reaches a segment on which it has already moved during a previous count ("wet" segment in the language of the classic Rainflow).

The Figure 1 shows an example of extracting a cycle (from point A to point I) from the load history A-B-C-D-E-F-G-H-I-J-K for a tensor having only the two shear components τ_{xz}, τ_{yz}.

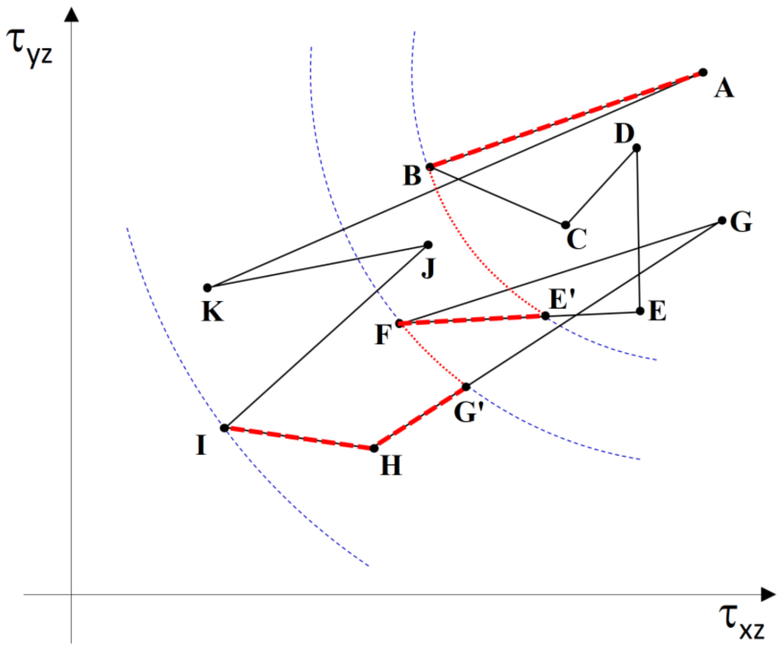

Figure 1 - Example of multiaxial cycle extraction

Observe from Figure 1 that moving from E to F the new point E' has been created in the sequence. The same happens when moving from G to H, when the new point G' is generated. This happens in the Rainflow multiaxial algorithm every time that $\sigma_{VM,REL}$ crosses and exceeds the maximum value of the current count.

M_4) A new cycle count starts from the second point of the sequence, and so on.

M_5) The process ends when the entire sequence has been covered.

At the end of the multiaxial cycle extraction process, all the segments of the stress history will have been counted once and only once, and all the cycles nested in larger loops will be recognized and extracted by the algorithm.
It is important to note explicitly that the cycles so extracted are actually half-cycles, or "inversions" in the language of fatigue, and not complete cycles: in the calculation of their damage made from a standard S-N curve given in terms of complete cycles, a factor ½ has to be applied.

CORRECTIONS OF THE STRESS AMPLITUDE

The multiaxial Rainflow illustrated in the previous section extracts from the load path in the stress space, cycles whose amplitude has not a unique definition. The simplest approach can be to define their amplitude σ_a through only the two initial and final points, let be σ_A and σ_B, in the following way:

$$\sigma_a = VM[(\underline{\underline{\sigma}}_A - \underline{\underline{\sigma}}_B)/2]$$

This agrees with the definition of σ_a given for linear cycles. Relatively to the Figure 1, these are the points A and I.

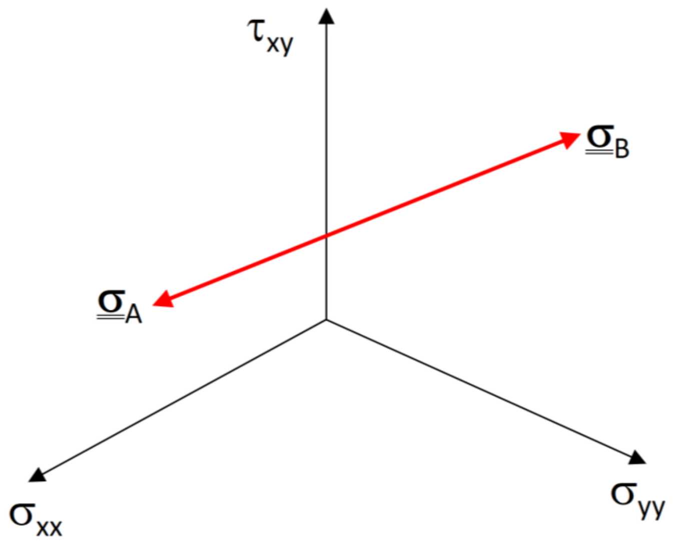

Figure 2 - Multiaxial linear cycle

In many operational applications, a so defined amplitude can be used to calculate the fatigue damage with an acceptable approximation.

However, since the work of Dang Van, Ref. [1], the researchers have developed several methodologies to improve this simple definition. Broadly speaking, an "equivalent amplitude" is defined, in a way that takes into account the actual load path. For example, let us consider the two paths (1) and (2) of Figure 3.

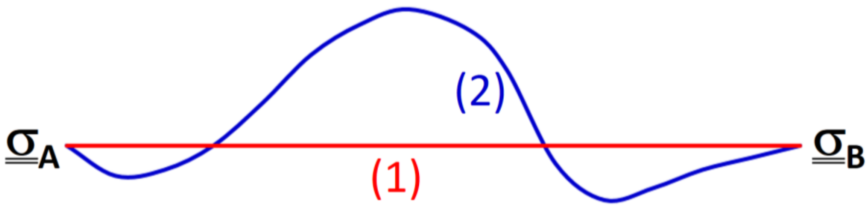

Figure 3 - Different stress paths with equal extremity points

Various approaches are available in the literature to take into account the non-proportionality of a load cycle and therefore to correct the amplitude of the cycles extracted with the multiaxial Rainflow based on the actual path of the stress: an exhaustive review is given for example in Ref. [4] and [5].

The corrections existing in the literature constitute an improvement with respect to the calculation of the amplitude σ_a carried out using only the points $\underline{\underline{\sigma}}_A$ and $\underline{\underline{\sigma}}_B$.

In a general way, they are related to geometric measurements in the space of deviatoric stresses: the minimum sphere defined by Dang Van, the minimum circumscribed ellipsoid, the prismatic hull, the longest chord, the moment of inertia (MOI) defined by Meggiolaro and Pinho de Castro, etc. : see for example Ref. [4] for a rigorous definition of such measures.

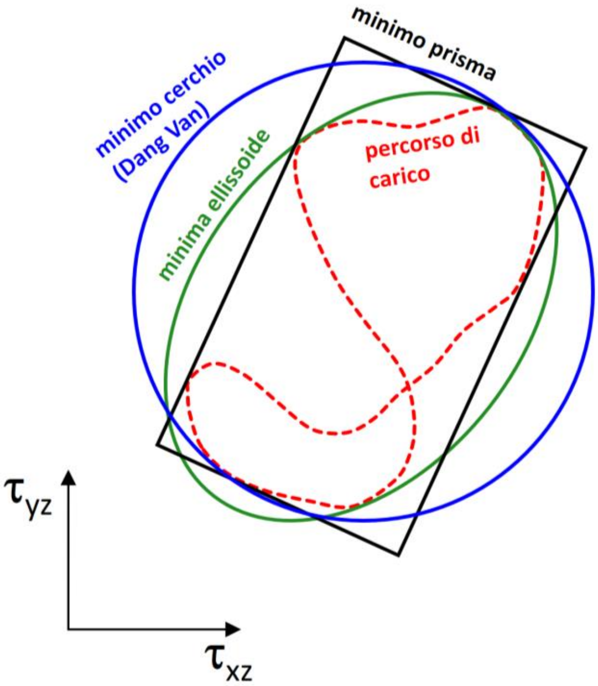

Figure 4 - Examples of amplitude correction of multiaxial cycles

However, a common aspect of all these corrections is that they inevitably constitute an approximation, as they do not fully highlight the effective multiaxial nature of the simplified cycles. This is illustrated through the following "critical" case.

Consider the stress path illustrated in Figure 5: the stress moves on points P1, P2, P3 and P4 as a function of a parameter λ which somehow defines how close the cycle is to the basic stress path obtained for $\lambda = 0$.

If $\lambda > 0$ the Rainflow multiaxial will extract two cycles, both of finite amplitude, while if $\lambda < 0$ only one cycle will be extracted.

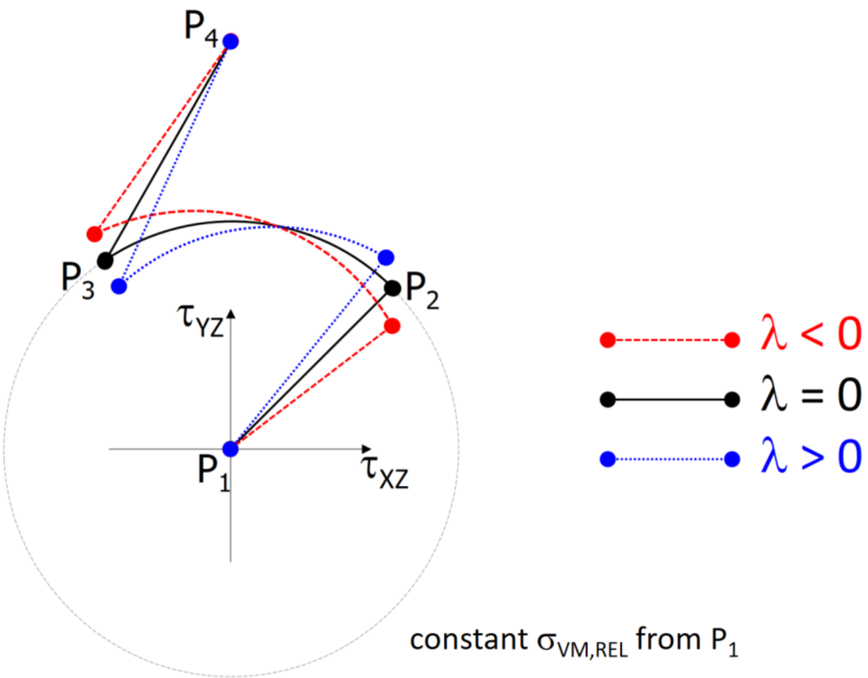

Figure 5 - Example of critical path

Therefore, when comparing $\lambda \rightarrow 0^+$ with $\lambda \rightarrow 0^-$ a physical incongruity can occur: in both cases the effort path $\lambda = 0$ can be approached at will, so the two options must rigorously provide the same result in terms of cycles extracted and therefore of damage calculation. However, it can be easily demonstrated that no "geometric" correction provides an exact solution.

The Method of Secondary Cycles (MSC), described here below, provides a more rigorous methodology.

The MSC does not use geometric corrections of the cycle extracted from the multiaxial Rainflow ("main cycle"), but introduces new load cycles extracted from the main one as follows:

C_1) The load history of the main cycle is projected in the subspace normal to the direction identified by the two end-points of the cycle itself, $\underline{\underline{\sigma}}_A$ and $\underline{\underline{\sigma}}_B$. Looking at the cycle of Figure 1, this is illustrated in the Figure 6 here below, where the blue cycle is the projection of the original one.

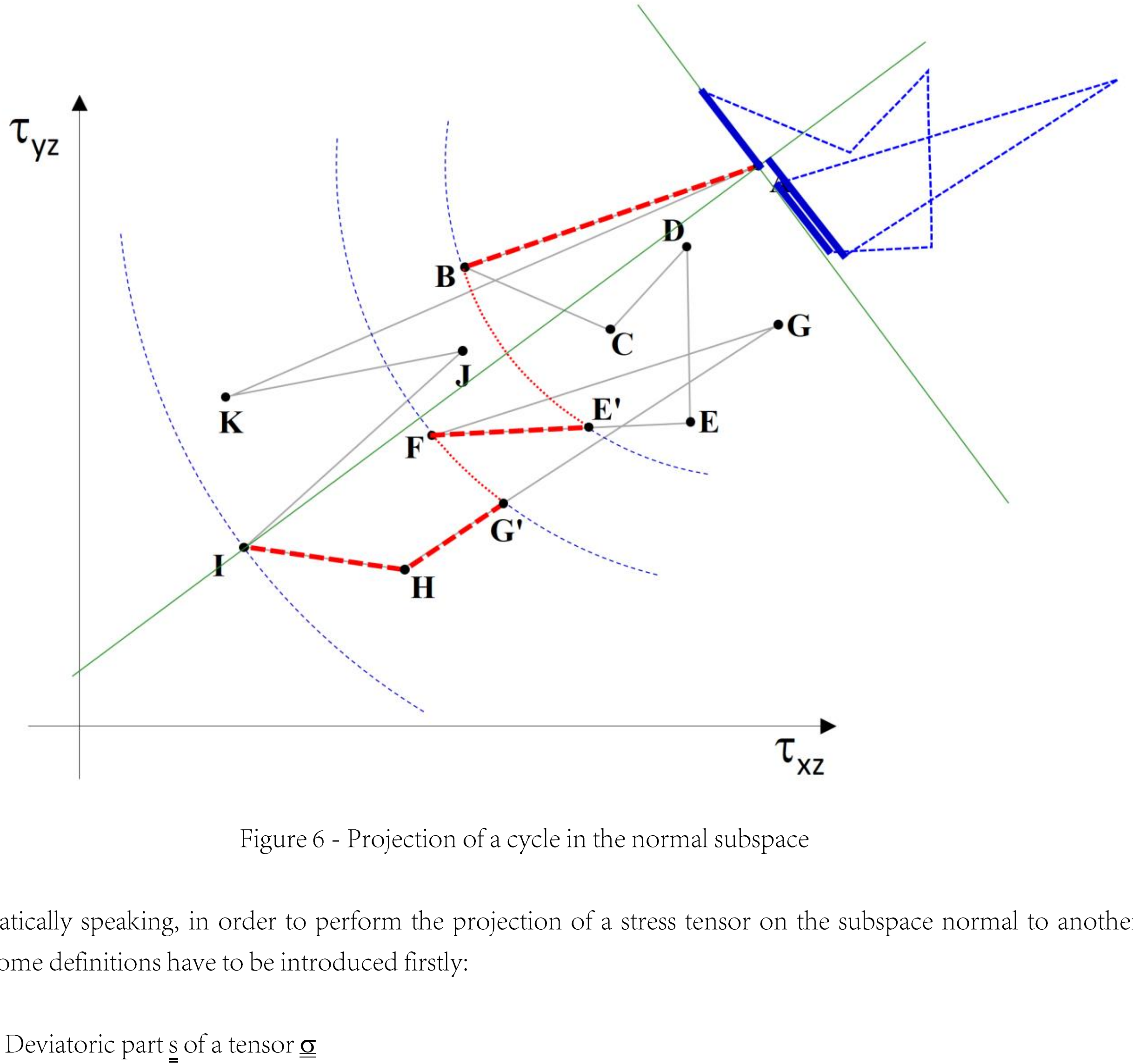

Figure 6 - Projection of a cycle in the normal subspace

Mathematically speaking, in order to perform the projection of a stress tensor on the subspace normal to another tensor, some definitions have to be introduced firstly:

DEF. 1 - Deviatoric part $\underline{\underline{s}}$ of a tensor $\underline{\underline{\sigma}}$

$$\underline{\underline{s}} = \text{dev}\, \underline{\underline{\sigma}} = \underline{\underline{\sigma}} - \text{tr}(\underline{\underline{\sigma}}) / 3\, \underline{\underline{I}}$$

where $\text{tr}(\underline{\underline{s}})$ is the trace or linear invariant: $\text{tr}(\underline{\underline{s}}) = s_{xx} + s_{yy} + s_{zz}$ and $\underline{\underline{I}}$ is the identity tensor.

DEF. 2 - Scalar product in the sense of Von Mises between the two purely deviatoric tensors $\underline{\underline{s}}_A$ and $\underline{\underline{s}}_B$

$$\underline{\underline{s}}_A : \underline{\underline{s}}_B = \sqrt{\frac{3}{2}} \sum_{i,j=1}^{3} s_{Aij} \cdot s_{Bij}$$

The Von Mises stress S_{VM} of a tensor $\underline{\underline{\sigma}}$ is

$$S_{VM}(\underline{\underline{\sigma}}) = \underline{\underline{s}} : \underline{\underline{s}}$$

DEF. 3 - Projection $\underline{\underline{p}}$ of a tensor $\underline{\underline{s}}_A$ on the subspace normal to $\underline{\underline{s}}_B$

$$\underline{\underline{p}}(\underline{\underline{s}}_A;\underline{\underline{s}}_B) = \underline{\underline{s}}_A - (\underline{\underline{s}}_A : \underline{\underline{s}}_B) / S_{VM}(\underline{\underline{s}}_B)$$

C_2) The new segments generated by the projection are included back in the sequence of stresses and a multiaxial rainflow process is restarted on the newly created sub-sequence starting from point $\underline{\underline{\sigma}}_A$ until the point $\underline{\underline{\sigma}}_B$.

C_3) Whenever a new cycle within the process C_1) and C_2) is created, if this new cycle is such that its scalar product with the segment $\underline{\underline{\sigma}}_A$ to $\underline{\underline{\sigma}}_B$ is positive, then this cycle too is projected on the subspace normal to ($\underline{\underline{\sigma}}_B$ - $\underline{\underline{\sigma}}_A$).

C_4) Every time a new cycle is extracted, it is treated via the same process described by the points C_1) and C_2).

C_5) Once this newly defined counting-process is completed, a further correction has to be applied in the frame of the MSC, namely, each extracted cycle has to be treated in the following way. Let us think to the sequence of stress-points that define the cycle and as an example let us consider the cycle in the Figure 7 below.

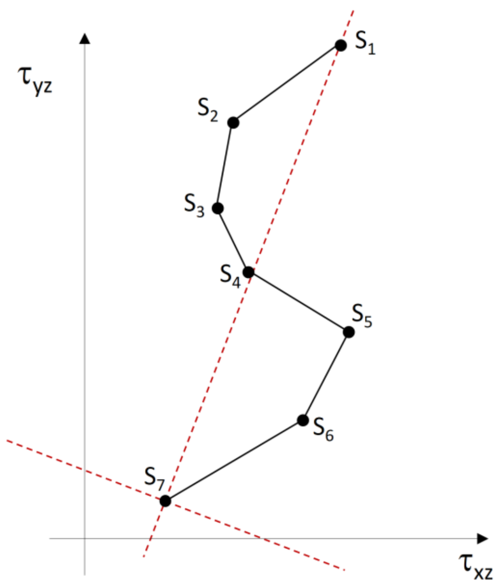

Figure 7 - Example of extracted cycle

Two types of points are defined in an extracted cycle:

Points of type "P1" are such that, if they are eliminated from the sequence of the extracted cycle, do not modify the projection of the cycle itself, defined at the point C_1) here above.

Points of type "P2" are such that, if they are eliminated from the sequence of the extracted cycle, the projection of the cycle itself, defined at the point C_1) here above, is somehow modified. We can define that the projection is modified by elimination of a stress-point, if a point exists in the projection after elimination of the stress-point, that does not belong to the projection made without eliminating the same stress-point, or if the number of such points on the projected segments is modified.

Referring to the cycle of Figure 6, the stress-points S_1, S_2, S_5 and S_7, are of type P2, whereas the stress-points S_3, S_4 and S_6 are of type P1.

Let us define the reduced extracted cycle, the one passing through the only points of type "P2" and having minimum length in the deviatoric stress-space, where, as a length of a segment in the stress space, the Von Mises stress of the difference between its extremes can be used. The length of a stress sequence will be simply the sum of the lengths of its segments. The reduction of the extracted cycle of Figure 6 is shown in blue in the Figure 8 here below.

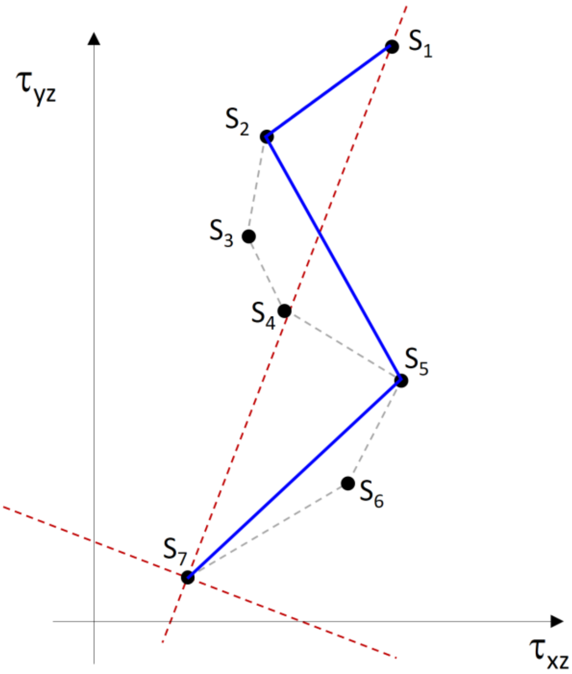

Figure 8 - Reduced extracted cycle

Once the reduced cycle is created, the stress-history formed by the original extracted cycle and its reduction has to be considered, in a way that takes the Von Mises stress of the segments formed by the difference if each point of the original cycle and its projection on the reduced one, along the direction of the segment between the first and the last points. The sign of the stress will be negative if the segment is oriented like the stress-segment from the first to the last point, positive in the other case. The Figure 9 here below illustrates the concept for the cycle of Figure 8. Note that in this case the signs are all positive.

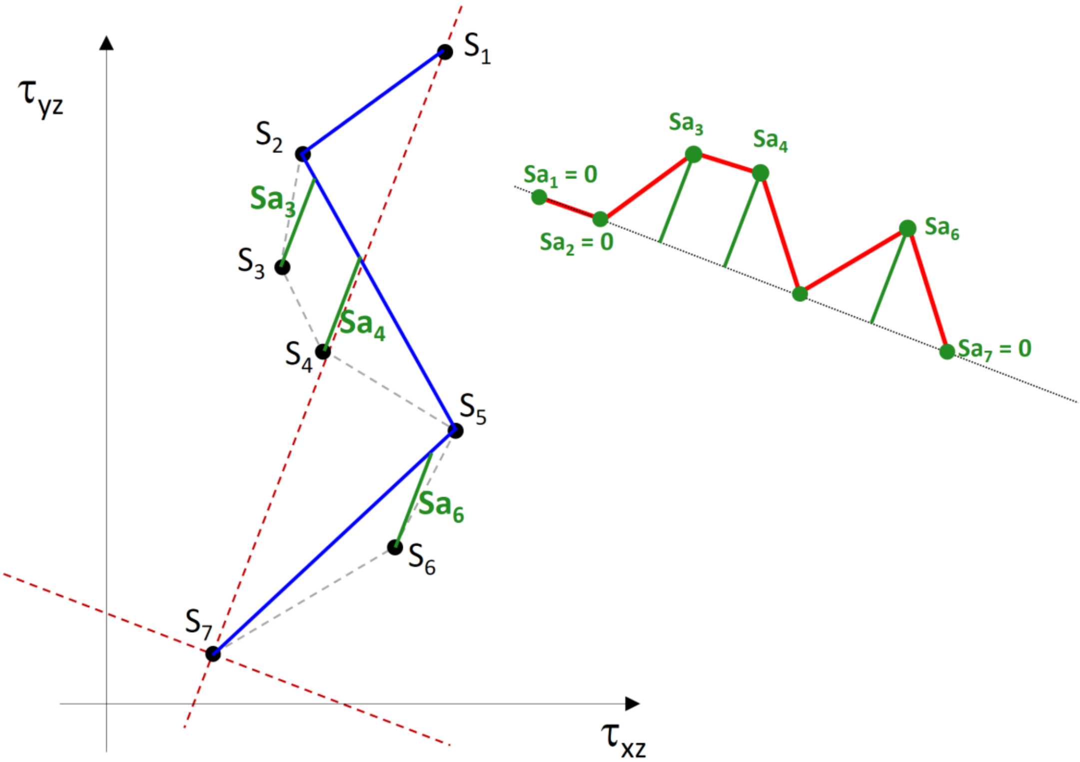

Figure 9 - Uniaxial stress-path built from the extracted cycle

The so-defined stress-history is uniaxial. A simple rainflow is then applied to this stress-history without reordering nor closing it, and the corresponding cycles are extracted: for the Figure 8 the result is the extraction of one full cycle with range Sa_4 and one full cycle with range Sa_6.

EXAMPLE OF APPLICATION OF THE METHOD

The MSC is conceived to avoid issues like the one illustrated by the critical path of Figure 5 above. To show this, the cycle itself of the Figure 5 is considered as an example. Let us think to start from the stress-point P1 and move up to the upper stress-point P4. The values of stress for the different points used in the example, are given in the Figure 10. As usual, for simplification only two shear components are considered.

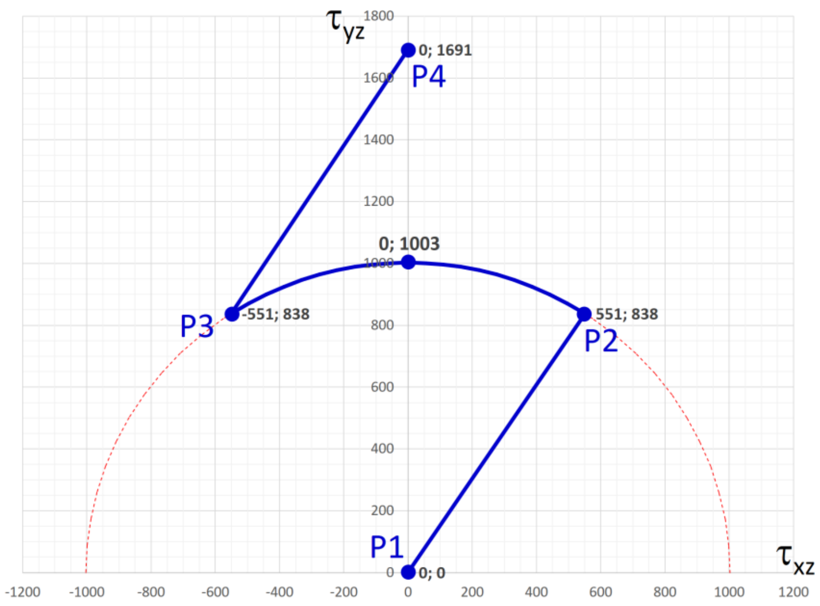

Figure 10 – Stress values for the application example

The considered path is P1, P2, P3, P4, P1. In the Table 1 here below, the amplitude of the cycles extracted via the Multiaxial Rainflow and with the MSC correction are shown, for the two cases $\lambda \to 0^+$ and $\lambda \to 0^-$.

$\lambda \to 0^+$	Sa	$\lambda \to 0^-$	Sa
c1	845.5	c1	845.5
c2	845.5	c2	845.5
c3	275.5	c3	275.5
c4	275.5	c4	275.5
c5	551.0	c5	551.0
c6	165.0	c6	165.0
c7	165.0	c7	165.0

Table 1 - Cycles generated with the MSC correction

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